e) Training

Essentially all affected supervisors were familiarized with the control procedures and purposes through personal contact and through the training courses described earlier. The transition from uncontrolled to controlled was scheduled over a period of one year. The procedures were first established in the Stress Unit and are now being extended to other technologies.

Results

Specific results from this type of effort are difficult to quantify. However, some trends and specific instances have been noted.

a) Coding errors occurred in about one out of every ten executions for the SAMECS program, a large finite element program used for substructure analysis of the 747 and SST airframes, prior to imposition of the controls. A few months after the imposition of the controls, the failure rate had dropped to about 1 coding error in 300 executions.

b) A comprehensive new solution module (about 25,000 source cards) was placed into the SAMECS program in July 1970. This solution module interfaced extensively with existing code and was validated using the method described in this paper. From July through November, only two coding errors occurred despite heavy using of between 250 and 350 executions each month.

c) A computer program developed under contract to NASA Langley was validated using the method described in this paper. The program has in the order of seven thousand executable statements. It executed on their computer installation without modification and has performed in daily to biweekly usage with only ten coding errors over a fifteen month period. Several new versions of the same program and additional programs have been delivered with the same result.

d) The program inventory was reduced by over 50%. This reduction released considerable tape and file storage.

e) Time formerly spent by programmers in counseling users and exercising "fire drills" has been reduced from around 50% of their total working time to around 10%.

f) The use of production engineering work to check out new programs or modifications to existing programs has been essentially eliminated.

g) Engineers are assured that they can return months afterwards to find a program still intact and possessing the same integrity as when last used.

h) One major operating system change has occurred. Programs under control were converted essentially without incident and without interruption to ongoing production work.

Conclusions

The procedures given in this paper have been effective in upgrading the reliability and quality of technical software within the Boeing Commercial Airplane Co. engineering staffs. Although care must still be exercised when committing production work to the computer, the amount of manpower and schedule time lost is only a fraction of the former condition. It would not be correct to imply that the improvements have come totally from these procedures. People and programs have matured and computer hardware has become more reliable. However, these procedures introduced discipline into the administration of program development, checkout, release, and modification. Significantly, the manpower required to control about 50 stress technology computer programs is about one quarter of one man. The procedures are largely self-administering, both engineers and programers having recognized the efficiency and convenience of the process.

Aside from the preceding, a principal benefit has been increased productively resulting from a better understanding of the computer by management and engineers, the removal of irrelevancies in engineering and programming work assignments, and a disciplined but unoffensive method of obtaining and maintaining computer program integrity.

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Incremented Flutter Analysis

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A method is presented to determine the magnitude of any particular increment (mass, stiffness, damping, etc.) necessary to satisfy prescribed flutter constraints. A reduced-order eigenvalue problem is formulated from the basic flutter equations, and the required increment is determined to the degree of accuracy inherent in the basic flutter equations. Application of the procedure to the evaluation of flutter with external stores is presented, as well as a simplified stiffness optimization procedure. Use of the method in an interactive (computer graphics) mode is described.

I. Introduction

DURING the design and analysis of an aircraft structural system, it is frequently desired to evaluate the effect of parameter changes on the flutter characteristics of the system. The modified parameters may be inertia, stiffness, aerodynamics, etc., and may represent a variety of external stores, stiffness configurations, mass ballast arrangements, or other items of interest. If the number of

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such parameter variations is large, it would obviously be desirable to cast the problem in a form requiring a minimum of elapsed time and computer capability. The method of Incremented Flutter Analysis makes use of the fact that many parameter variations of practical interest may be described in terms of a relatively small number of coordinate degrees of freedom. In addition, it is frequently more useful to define the effect of the incremented parameter in terms of the required or allowed magnitude which just satisfies prescribed flutter constraints, rather than to determine the effect of a given increment of the parameter. One example of this is the problem of determining the characteristics of a family of external stores which results in acceptable flutter margins.

The method was formulated for use in conjunction with a finite element structural analysis employing a reasonably large number of explicit structural degrees of freedom, and the advantages of the method are greatest when used in this context. It will be shown, however, that the method can be adapted to analyses employing generalized coordinates, and in fact is quite versatile in that it can be used with a wide variety of basic flutter analysis procedures. As indicated earlier, the method of Incremented Flutter Analysis is generally formulated in terms of a reduced number of degrees of freedom compared with the basic flutter analysis. It should be emphasized that this reduction of the basic flutter equations retains all of the characteristics of the original flutter equations; the only approximations involved are those resulting from inherent computational limitations.

II. Method and Derivation of Equations

A. Basic Concept

In order to demonstrate the concept of Incremented Flutter Analysis, consider a flutter equation of the general form where q are the displacements, p is the nondimensional operator, and M, K, and A are the matrices of coefficients of mass, stiffness, and aerodynamics. Although the aerodynamic parameters in Eq. (1) are indicated to be functions of the time derivative p, it will be seen that aerodynamic parameters as functions of reduced frequency are equally acceptable, since these parameters are evaluated at discrete frequencies in the ensuing procedures. For the present, the coordinates q will be treated as explicit structural coordinates, and the treatment of generalized coordinates will be considered later.

$$\left[[M] \frac{V^2}{C^2} p^2 + [K] - 1/2\, \rho V^2 [A(p)] \right] \{q\} = 0 \tag{1}$$

The terms of Eq. (1) are replaced by the condensed notation of Eq. (2), and this latter equation is taken to represent the basic system which is to be modified. Equation (3) then represents the basic system modified by the addition of an increment of the parameter of interest. This parameter may be any quantity (mass, stiffness, aerodynamics, etc.) suitable for inclusion in the original flutter equations.

$$[D(p)]\{q\} = 0 \tag{2}$$

$$\left[[D(p)] + [\Delta D(p)] \right] \{q\} = 0 \tag{3}$$

In most cases, the incremented parameter can be defined in terms of a subset of the total degrees of freedom of the system. As an illustration, the addition of a point mass might be defined in terms of a single degree of freedom of the base system. Accordingly, Eq. (3) is partitioned into two subsets of equations [Eq. (4)] corresponding to those coordinates necessary to define the increment (q_1) and the remaining coordinates (q_2).

$$\left[\left[\begin{array}{c|c} D_A(p) & D_B(p) \\ \hline D_C(p) & D_D(p) \end{array} \right] + \left[\begin{array}{c|c} \Delta D(p) & 0 \\ \hline 0 & 0 \end{array} \right] \right] \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = 0 \tag{4}$$

$$[D_A(p)]\{q_1\} + [D_B(p)]\{q_2\} + [\Delta D(p)]\{q_1\} = 0 \tag{5}$$

$$[D_C(p)]\{q_1\} + [D_D(p)]\{q_2\} = 0 \tag{6}$$

$$\{q_2\} = -[D_D(p)]^{-1}[D_C(p)]\{q_1\} \tag{7}$$

$$[D_A(p) - D_B(p) D_D(p)^{-1} D_C(p)]\{q_1\} + [\Delta D(p)]\{q_1\} = 0 \tag{8}$$

This partitioned equation is next expanded into the separate subsets [Eqs. (5) and (6)] of the incremented and nonincremented coordinates. Equation (7) expresses the nonincremented coordinates q_2 in terms of the incremented coordinates q_1. This relationship is substituted into Eq. (5) to eliminate the coordinates q_2, resulting in the reduced system of equations shown in Eq. (8). This procedure will be recognized as the equivalent of the more familiar stiffness matrix reduction,[1] and results in reducing the incremented flutter equation to the coordinates just sufficient to define the particular increment.

It will be recognized that, except for trivial cases, the operation indicated by the matrix inverse in Eqs. (7) and (8) cannot be formulated directly. However, the equations can be evaluated at discrete values of the frequency parameter p for a specified value of airspeed and density. This is expressed in Eq. (10), where the condensed notation of Eq. (9) is used to represent the reduced equations of the base system.

$$[DR(p)] \equiv [D_A(p) - D_B(p) D_D(p)^{-1} D_C(p)] \tag{9}$$

$$[DR(p_i)]\{q_1\} + [\Delta D(p_i)]\{q_1\} \neq 0 \qquad i = 1, 2, \ldots n \tag{10}$$

$$[DR(p_i)]\{q_1\} + \lambda[\Delta D(p_i)]\{q_1\} = 0 \qquad i = 1, 2, \ldots n \tag{11}$$

Since all the coefficients of Eq. (10) are specified, the determinant of these coefficients will not vanish for any arbitrary value of the frequency parameter p_i. For any specified value of the frequency parameter, however, there exists a scalar multiplier which, when applied to the increment, will cause the determinant of the coefficients to vanish. This is expressed in Eq. (11), which is seen to be in the familiar form of a characteristic value (eigenvalue) problem, and which may be solved by any of the usual methods applied to such problems. At each value of the frequency parameter, then, a value of the scalar multiplier may be determined which satisfies the equation. In view of the physical meaning of the equation, this scalar multiplier represents the magnitude of the increment which, at that particular frequency, produces a neutrally stable flutter mode. In general, this scalar multiplier will be complex, indicating that no real value of the increment will produce neutrally stable flutter at that particular frequency and airspeed. If the imaginary part of the scalar goes to zero within the frequency range of interest, the real part of the scalar represents the magnitude of the increment which will just satisfy the flutter constraints implied by the parameters (frequency, airspeed, density) used to evaluate the coefficients.

Since the coefficients of Eq. (11) are evaluated at discrete frequencies, the scalar multiplier is determined at these same frequencies. The frequency at which the imaginary part of the multiplier vanishes, and the corresponding value of the real part of the multiplier, must be determined by an approximate means using curve-fitting and/or interpolation procedures. This approximation can be controlled to any desired degree of accuracy, however, by the frequency interval chosen for the evaluation of the equation.

B. Procedure

Although the formulation of the equations in Sec. II-A serves to illustrate the concept of Incremented Flutter Analysis, the form of the equations is not particularly convenient for actual use. In the form presented, the reduced flutter equation would have to be reformulated for each increment involving a different set of coordinates, necessitating many repetitions of the matrix inverses indicated in Eq. (8). A more convenient form results from premultiplying Eq. (11) by the inverse of the first matrix of that equation, resulting in the form shown in Eq. (12). This latter inverse can be shown to be identical to the corresponding subset of the inverse of the complete matrix of coefficients of the base system, indicated in Eqs. (13–15). Just as the original formulation is analogous to the reduction of the stiffness matrix, the present formulation is analogous to inverting the complete stiffness matrix and extracting the reduced flexibility matrix. With this procedure, any number of coordinate subsets can be incremented by extracting the appropriate subsets from the inverses of the coefficients of the base flutter system and using these in Eq. (15)

$$[[1] + \lambda[DR(p_i)]^{-1}[\Delta D(p_i)]]\{q_1\} = 0 \tag{12}$$

$$[DR(p_i)]^{-1} \equiv [DI_A(p_i)] \tag{13}$$

$$[D(p_i)]^{-1} \equiv \left[\begin{array}{c|c} DI_A(p_i) & DI_B(p_i) \\ \hline DI_C(p_i) & DI_D(p_i) \end{array}\right] \tag{14}$$

$$[[1] + \lambda[DI_A(p_i)][\Delta D(p_i)]]\{q_1\} = 0 \tag{15}$$

To recapitulate, the procedure is implemented as follows:

1) The matrix of coefficients of the base flutter system is evaluated at discrete frequencies over the range of interest and the inverses of these matrices obtained.

2) From the matrices obtained in 1), the reduced matrices are formed corresponding to the coordinates to be incremented.

3) These reduced matrices are used with the desired increment to form the set of eigenvalue problems indicated by Eq. (11).

4) By graphical, curve-fitting, or interpolation methods, the real part of the eigenvalue is determined at the frequency at which the imaginary part of the eigenvalue vanishes. The real part of the eigenvalue is the magnitude of the increment which results in flutter at exactly the condition for which the flutter coefficients were evaluated.

C. Successive Increments

In some applications, it may be desirable to modify the basic flutter equations by a prescribed amount of one increment and then to determine the magnitude of a different increment which will just satisfy flutter contraints. The obvious method, of course, is to add the first increment to the base flutter equations, treat the resultant as a new base case, and proceed as before. If the subset of coordinates needed to describe both increments is significantly less than the total number of coordinates, the eigenvalue problem can be formulated as in Eq. (16), where the reduced inverses (DI_A) correspond to the total number of coordinates required to define both increments. For efficiency of computation, an alternate form may be used wherein a general subset of the flutter coefficient inverses is updated, and the specific subset needed for the next increment is formed from these. In Eq. (17), the matrices DI_A correspond to the complete subset of the flutter coefficient inverses required to define a series of increments. The left-hand side of this equation now represents an updated, reduced set of flutter coefficient inverses from which a small subset may be formed corresponding to the next increment to be evaluated. It should be noted that both the updating of the flutter coefficient inverses and the formulation of the eigenvalue problem should be implemented using the minimum coordinate subset necessary. In addition to the obvious reasons of economy of computation, minimizing the order of the eigenvalue problem eliminates roots which are of no interest. Although these roots are easily distinguishable from physically significant roots, they do create a nuisance. The subject of minimizing the order of the eigenvalue problem will be treated further in Sec. II-D.

$$[[1] + [DI_A(p_i)][\Delta D_1(p_i)] + \lambda[DI_A(p_i)][\Delta D_2(p_i)]]\{q_1\} = 0 \tag{16}$$

$$[D'(p_i)]^{-1} \equiv [[1] + [DI_A(p_i)][\Delta D_1(p_i)]]^{-1}[DI_A(p_i)] \tag{17}$$

D. Transformations, Modal Equations

Once the inverses of the flutter coefficient matrices are obtained, any desired coordinate transformation may be performed. If the relationship between the original coordinates (q) and the new coordinates (q') is indicated by Eq. (18), the flutter coefficient inverses in the new coordinate system are defined by Eq. (19). Such transformations are frequently used to generate a set of coordinates more suited to the specific increment to be studied and thus minimize the order of the eigenvalue problem. As an example, assume the simple torque tube element in Fig. 1 to be the

$$\{q'\} = [T]\{q\} \tag{18}$$

$$[D'(p_i)]^{-1} = [T][D(p_i)]^{-1}[T]^T \tag{19}$$

incremented element. In the Θ, ϕ coordinate system, the stiffness matrix of this torque tube requires four coordinates and the eigenvalue problem would be of fourth order. Transformed to the Θ', ϕ' system, however, only two coordinates are required, resulting in a second-order eigenvalue problem. If the transformation is to the relative coordinate $\Theta_2' - \Theta_1'$ (the elemental strain coordinate of the torque tube), the stiffness and therefore the eigenvalue problem can be formulated in terms of a single coordinate.

With the increasing use of finite-element structural analysis methods, the structural representations available for use in flutter analyses have become extremely sophisticated, with very many degrees of freedom. In order to use these structural models in a flutter analysis, the number of degrees of freedom must usually be reduced by means of elimination, reduction, and/or modalization. In the usual case, the generalized coordinates $\bar{q}$ are related to the discrete coordinates q through a partial set of the vibration vectors [Eq. (20)] and the matrix of coefficients of the flutter equation is modalized as in Eq. (21). In applying the present method to this form of the flutter equation, the modalized inverses are obtained and the unmodalized

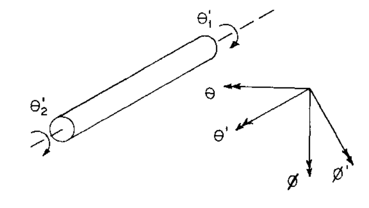

Fig. 1 Torque tube element.

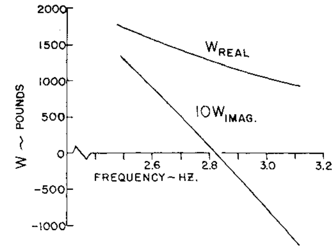

Fig. 3 Weight scalar vs frequency.

inverses are approximated as indicated in Eq. (22); the method then proceeds as before. It has been found that the errors resulting from this approximation are small if the number of modes is sufficient to represent the incremented system.

$$\{q\} = [V]\{\bar{q}\} \tag{20}$$

$$[\bar{D}(p_i)] = [V]^T[D(p_i)][V] \tag{21}$$

$$[D(p_i)]^{-1} \approx [V][\bar{D}(p_i)]^{-1}[V]^T \tag{22}$$

Applications

Mass Ballast

The use of the present procedure for the determination of the mass ballast is a particularly simple application of the method, and some of the details of the test case performed to verify the method will be presented. The base flutter system was a simple finite element beam representation of a wing-body having a total of 67 elastic degrees of freedom, including 9 bending and 9 torsion degrees of freedom on the wing semispan. The torsional stiffness of the wing was reduced in order to produce a flutter mode within the speed range of the airplane, and the flutter analysis of this configuration gave the bending-torsion flutter characteristics shown in Fig. 2. Since the frequency of the flutter mode was approximately 3.5 Hz, the matrix of flutter coefficients was evaluated at an airspeed of V_0 for frequencies from 2.5–3.5 Hz, and the inverses of these matrices obtained. These inverses were next transformed to generate vertical deflection coordinates forward and aft of the wing elastic axis while retaining the original 9 vertical deflections on the elastic axis. In principle, the diagonal element corresponding to the specific coordinate in question was extracted from each inverse, and a first-order eigenvalue problem formulated as in Eq. (23)

$$1 + \lambda(DI_A(p_i))m\frac{C^2}{V^2}p_i^{\,2} = 0 \tag{23}$$

where m is any convenient reference mass. In practice, all 27 ballast locations were analyzed in a single formulation by solving for a diagonal matrix of eigenvalues. Not surprisingly, the most efficient ballast location of those analyzed was the forward location at the tip station. The eigenvalue for this location is shown in Fig. 3 for the frequency range investigated, indicating that a weight of 1255 lb would result in a flutter crossing at V_0 in a 2.82 Hz mode. Adding this increment to the base flutter system and analyzing this configuration results in the modified flutter root shown in Fig. 4. The results of this test case, and comparable test cases for other applications, indicate that the accuracy of the method is comparable to the computational accuracy of the basic analysis.

External Stores

The flutter analysis of external stores is perhaps the application which best demonstrates the potential and versatility of the method. For simplicity of discussion, assuming the base flutter analysis to be formulated for a wing without external stores, the effect of a rigid store attached in bending and torsion at the tip station is evaluated. As before, the matrix of flutter coefficients is evaluated at the required flutter speed and at frequencies throughout the range of interest, and the matrix inverses obtained. From the inverses, the 2 × 2 submatrices corresponding to the tip bending and torsion cordinates are generated, and these are the DI_A matrices indicated in Eq. (15). The general form of the increment representing the external store, including aerodynamics, is given in Eq. (24), where d is the distance of the store center of gravity from the reference axis. If it is desired to determine the weight of a store of specified center of gravity and radius of gyration which will just flutter at the reference velocity, Eq. (25) is used, where the eigenvalue is an over-all factor on the total store inertia. Equation (26) is used if it is desired to specify the weight and center of gravity of the store and then determine the allowable pitching moment of inertia about the center of gravity. As another alternative, the form indicated in Eq. (27) represents a store of fixed weight and radius of gyration and the eigenvalue represents the allowable center of gravity limit.

$$[\Delta D(p_i)] = \left[\begin{array}{c|c} m & md \\ \hline md & md^2 + I_0 \end{array}\right]\frac{V^2}{C^2}p_i^{\,2} - 1/2\,\rho V^2[A_S(p_i)] \tag{24}$$

$$[\Delta D(p_i)] = -1/2\,\rho V^2[A_S(p_i)] + \lambda\left[\begin{array}{c|c} m & md \\ \hline md & md^2 + I_0 \end{array}\right]\frac{V^2}{C^2}p_i^{\,2} \tag{25}$$

$$[\Delta D(p_i)] = -1/2\,\rho V^2[A_S(p_i)] + \left[\begin{array}{c|c} m & md \\ \hline md & md^2 \end{array}\right]\frac{V^2}{C^2}p_i^{\,2} + \lambda\left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & I_0 \end{array}\right]\frac{V^2}{C^2}p_i^{\,2} \tag{26}$$

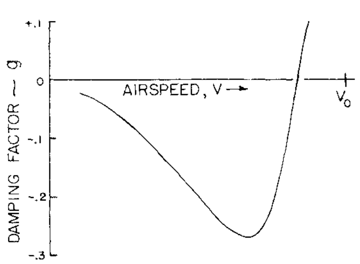

Fig. 2 Bending-torsion flutter mode.

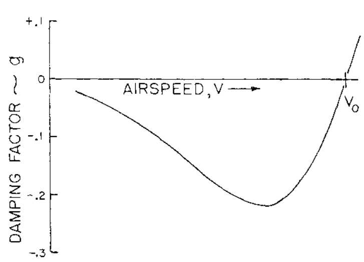

Fig. 4 Bending-torsion flutter mode with 1255-lb ballast.

$$[\Delta D(p_i)] = -1/2\,\rho V^2 [A_S(p_i)] + \left[\begin{array}{c|c} m & 0 \\ \hline 0 & I_0 \end{array}\right] \frac{V^2}{C^2} p_i^2 + \lambda \left[\begin{array}{c|c} 0 & m \\ \hline m & 0 \end{array}\right] \frac{V^2}{C^2} p_i^2 + \lambda^2 \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & m \end{array}\right] \frac{V^2}{C^2} p_i^2 \quad (27)$$

Using one or more of these forms, a set of flutter boundaries can be derived which define the complete range of store inertia characteristics which will satisfy the given flutter constraints. It should be noted that these small-order eigenvalue problems can be solved very rapidly and economically using a wide range of computing equipment including remote access computers and some desk-top computers. The formulations of the eigenvalue problem considered here are by no means complete; the form is readily adaptable to the desired analysis. For instance, Eq. (28) shows a formulation which could be used to solve for the level of aerodynamic forces (such as the forces produced by an aerodynamic vane) required with a specific tip store to satisfy flutter constraints. Although not shown here, the extension to include flexibly mounted stores is easily derived.

$$[D(p_i)] = \left[\begin{array}{c|c} m & md \\ \hline md & md^2 + I_0 \end{array}\right] \frac{V^2}{C^2} p_i^2 - 1/2\,\rho V^2 \lambda [A_S(p_i)] \quad (28)$$

Stiffness Optimization

Although several methods of structural optimization for flutter are available (2,3,4,5,6,7,8, etc.), the present method can be used in a procedure which differs in some respects from existing methods. Rather than using the derivative of flutter speed with respect to the design variables, as many methods do, the increment in each design variable necessary to satisfy the flutter constraints is determined directly. Based on this result, stiffness material is distributed in such a way as to incur the minimum weight penalty.

As a simplified example of the procedure, assume the same base flutter case used previously to illustrate the mass ballasting procedure, i.e., a flutter analysis indicating a flutter speed deficiency in the bending-torsion flutter mode, the structural representation being a simple beam model as before. Once again, the matrix of flutter coefficients is evaluated at the appropriate speed and frequencies, and the inverses obtained. Since it is desired to determine the optimum torsional stiffness distribution, the inverses are transformed to obtain the strain coordinates of the torsional stiffness elements, resulting in one coordinate for each stiffness element.

In the manner previously indicated, a first-order eigenvalue problem is formulated for each torsional stiffness element, the increment in that element required to satisfy the flutter constraints is determined, and the element requiring the minimum-weight increment is identified. Stiffness is added to this element and, within the limitations of the sizing constraints, removed from the least efficient element. This modified stiffness distribution is used to update the subset of inverses as in Eq. (17). This procedure is continued until all design variables, except those limited by sizing constraints, require an equal weight increment to satisfy the flutter constraints, at which point no further weight reduction is possible. It should be noted that this simplified procedure does not account for the inertia increment associated with the stiffness increment, and as such is appropriate for a preliminary design procedure wherein the weight is periodically updated. If it is desired to include the inertia effects during each step, the incremented parameter is merely defined so as to include both the stiffness and inertia characteristics of the unit design variable, usually at the expense of an increase in the number of coordinates required to define the increment.

Interactive Mode

The mass ballasting and external stores procedures presented earlier are well adapted to a batch processing mode of computer operation, since the detailed analytical procedure can be specified in advance. The suggested stiffness optimization procedure, however, can be implemented very advantageously in an interactive mode, particularly one which employs computer graphics. The formulation of the base flutter case and the generation of the matrix inverses can probably best be done in the batch computing mode, but the solution of the small-order eigenvalue problems, the determination of the real roots, and any updating of the subsets can be done much more efficiently using the smaller computing capacities and quick turn-around capabilities of a remote access/interactive facility. For the iterative parts of the stiffness optimization, an interactive capability is virtually a necessity.

Conclusion

The method of Incremented Flutter Analysis can be used to great advantage when 1) the incremented parameters can be defined using relatively few coordinates, 2) the number of parameter variations applied to each base configuration is large, 3) the flutter criteria may be expressed in terms of a small number of critical speed/altitude combinations, and 4) a flutter analysis of the base configuration exists, providing some background regarding the flutter characteristics of the system. It should be noted that the absence of any of these conditions does not preclude the use of the method, but merely reduces the advantage of the method relative to conventional flutter analysis.

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